

Date Completed: \_\_\_\_\_  
Mentor Initials: \_\_\_\_\_

A mentor can change everything.



## Number Properties

1. What is the greatest common factor of 32, 56, and 96?
  - A. 2
  - B. 4
  - C. 6
  - D. 7
  - E. 8
2. What is the smallest integer greater than  $\sqrt{24}$ ?
  - A. 4
  - B. 5
  - C. 6
  - D. 7
  - E. 8
3. What is the smallest positive integer having exactly 6 different positive integer divisors?
  - A. 4
  - B. 5
  - C. 6
  - D. 10
  - E. 12
4. Consider all positive integers that are multiples of 12 and that are less than or equal to 200. What fraction of those integers are multiples of 8?
  - A.  $\frac{1}{5}$
  - B.  $\frac{1}{4}$
  - C.  $\frac{1}{3}$
  - D.  $\frac{1}{2}$
  - E.  $\frac{2}{3}$
5. What is the remainder when 792 is divided by 7?
  - A. 0
  - B. 1
  - C. 2
  - D. 3
  - E. 5

6. Given consecutive positive integers  $a, b, c,$  and  $d$  such that  $a < b < c < d$ , which of the following expressions has the least value?
- A.  $\frac{a}{b}$
  - B.  $\frac{b}{c}$
  - C.  $\frac{c}{d}$
  - D.  $\frac{a+b}{b+c}$
  - E.  $\frac{b+c}{c+d}$
7. The least common multiple (LCM) of 2 numbers is 192. The larger of the 2 numbers is 96. What is the greatest value the other number can have?
- A. 4
  - B. 8
  - C. 24
  - D. 64
  - E. 74
8. If both  $x$  and  $\left(\frac{x}{4} + \frac{x}{7} + \frac{x}{8}\right)$  are positive integers, what is the least possible value of  $x$ ?
- A. 24
  - B. 32
  - C. 48
  - D. 56
  - E. 72
9. What is the product of the mean and the median of the first seven prime numbers? (Note: 2 is the first prime number.)
- A. 31
  - B. 43
  - C. 58
  - D. 63
  - E. 72

10. The digit in the ones place of  $2^{65}$  is 2. What is the digit in the ones place of  $2^{81}$ ?
- A. 0
  - B. 2
  - C. 4
  - D. 6
  - E. 8
11. Henry and Anika start cycling laps from the same starting line at the same time and in the same direction on their favorite loop outside the city. Henry completes one lap in 17 minutes, and Anika completes the same lap in 14 minutes. Both continue cycling at their same respective rates and in the same direction for 4 hours. What is the fewest number of minutes after starting that Henry and Anika will again be at their starting line at the same time?
- A. 96
  - B. 119
  - C. 170
  - D. 210
  - E. 238
12. For some positive integer  $q$ , the sum of the absolute values of all the integers from  $-q$  through  $q$  is 20. What is  $q$ ?
- A. 2
  - B. 3
  - C. 4
  - D. 5
  - E. Cannot be determined from the given information
13. Which of the following expressions, when evaluated, equals an irrational number?
- A.  $\frac{\sqrt{3}}{\sqrt{27}}$
  - B.  $\frac{\sqrt{27}}{\sqrt{3}}$
  - C.  $(\sqrt{27})^2$
  - D.  $\sqrt{3} + \sqrt{27}$
  - E.  $\sqrt{3} \times \sqrt{27}$

14. Given  $0 < x \leq 20$  and  $y \geq 16$ , what is the greatest value of  $\frac{x-y}{y}$ , if it can be determined?
- A. 0
  - B.  $\frac{1}{4}$
  - C. 1
  - D.  $\frac{15}{4}$
  - E. Cannot be determined from the given information.
15. Let  $a$  and  $b$  represent real numbers with the property that  $|a + b - 3| > 2$ . Which of the following statements about  $a$  and  $b$  CANNOT be true?
- A.  $a + b < 1$
  - B.  $a + b = 5$
  - C.  $a > 4$  and  $b > 1$
  - D.  $a > 2$  and  $b > 3$
  - E.  $a < 0$  and  $b < 1$
16. There are 720 students in the local high school, and they are all lined up to be counted. The principal decides to start on a random student and count off every  $n$  students, where  $n$  is an integer, and wishes to make sure every student is counted. What could be the integer that the principal chose?
- A. 5
  - B. 6
  - C. 7
  - D. 8
  - E. 9

17. Elisa is inventing a new notation in math. She has decided to use  $m \ominus$ , where  $m$  is always an integer, to denote the operation of summing half of each integer from 1 to  $m$ . For example,  $3 \ominus$  means  $\frac{1}{2} + \frac{2}{2} + \frac{3}{2}$ . Elisa has written 3 statements that she is investigating as possible properties of  $m \ominus$ .

- I.  $(m + 1) \ominus = m \ominus + 1 \ominus$   
II.  $(2m) \ominus = m \ominus + m \ominus$   
III.  $(m + 1)^2 \ominus = m^2 \ominus + m \ominus + 1 \ominus$

Which of these statements, if any, is (are) true for all positive integers  $m$ ?

- A. I only  
B. II only  
C. III only  
D. I, II, and III  
E. None
18. The greatest common factor of two whole numbers is 2. The least common multiple of these same two numbers is 276. What are the two numbers?
- A. 14 and 32  
B. 12 and 34  
C. 12 and 46  
D. 40 and 50  
E. 44 and 60
19. Which of the following most precisely describes the roots of the equation  $7x^2 + 2x - 4$ ?
- A. 1 rational double root  
B. 1 irrational double root  
C. 2 distinct rational roots  
D. 2 distinct irrational roots  
E. 2 complex roots (with nonzero imaginary parts)
20. The number 5,083 is the product of the prime numbers 13, 17, and 23. What is the prime factorization of 188,071?
- A.  $2 \cdot 13 \cdot 17 \cdot 23$   
B.  $13 \cdot 17 \cdot 23 \cdot 37$   
C.  $2 \cdot 13 \cdot 17 \cdot 23 \cdot 37$   
D.  $3 \cdot 7 \cdot 13 \cdot 17 \cdot 23 \cdot 37$   
E.  $2 \cdot 3 \cdot 7 \cdot 13 \cdot 17 \cdot 23 \cdot 37$